Chapter 4

Moving Charges and Magnetism

Magnetic Field & Biot Savart Law

Magnetism:

1. The Magnetic Field:

In earlier lessons we found it convenient to describe the interaction between charged objects in terms of electric fields. Recall that an electric field surrounding an electric charge. The region of space surrounding a moving charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds a magnetic substance.
In order to describe any type of field, we must define its magnitude, or strength, and its direction.

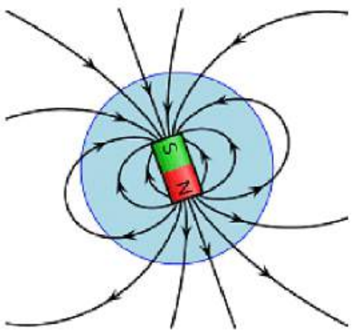


Fig: Magnetic field

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Magnetic field intensity is a vector quantity and also known as magnetic induction vector. It is represented by $\vec{B}$
Lines of magnetic induction may be drawn in the same way as lines of electric field. The number of lines per unit area crossing a small area perpendicular to the direction of the induction being numerically equal to $\vec{B}$. The number of lines of $\vec{B}$ crossing a given area is referred to as the magnetic flux linked with that area. For this reason $\vec{B}$ is also called magnetic flux density.

There are two methods of calculating magnetic field at some point. One is **Biot-Savart law** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere' law,** which is useful in calculating the magnetic field of a symmetric configuration carrying a steady current.

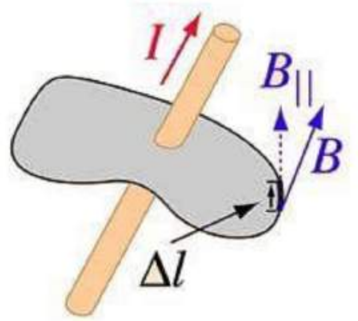


Fig: Ampere's law

The unit of magnetic field is weber/m^2 and is known as tesla (T) in the SI system.

2. BIOT-SAVART LAW:

Biot-Savart law gives the magnetic induction due to an infinitesimal current element.

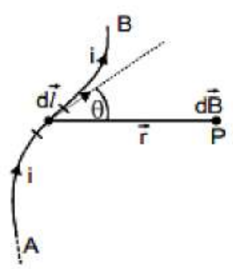
Let AB be a conductor of an arbitrary shape carrying a current i, and P be a point in vacuum at which the field is to be determined. Let us divide the conductor into infinitesimal current-elements. Let $\vec{r}$ be a displacement vector from the element to the point P.

According to `Biot-Savart Law', the magnetic field induction at P due to the current element $d\vec{l}$ is given by

$d\vec{B} \propto \frac{i(d\vec{l} \times \vec{r})}{r^3}$ or $d\vec{B} = k\frac{i(d\vec{l} \times \vec{r})}{r^3}$

Where k is a proportionality constant.

Here $d\vec{l}$ vector points in the direction of current i.



In S.I. units,

$$\therefore d\vec{B} = \frac{\mu_0}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3}$$

Equation (1) is the vector form of the Biot-Savart Law. The magnitude of the field induction at P is given by:

$$dB = \frac{\mu_0}{4\pi}\frac{idl \times \sin\theta}{r^2}$$

where θ is the angle between $d\vec{l}$ and $\vec{r}$.
If the medium is other than air or vaccum, the magnetic induction is:

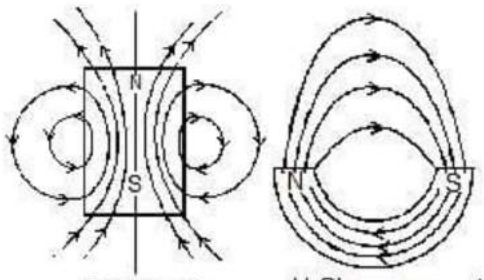
$$d\vec{B} = \frac{\mu_0\mu_r}{4\pi}\frac{i(d\vec{l} \times \vec{r})}{r^3}$$

where μ_r is relative permeability of the medium and is a dimensionless quantity.

Magnetic Lines & Their Characteristics

6. Magnetic Lines and Their Characteristics

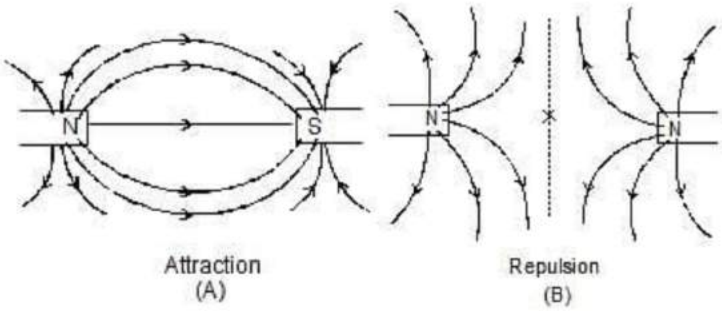
The space surrounding a magnet or magnetic configuration in which its effects are perceptible is called the **magnetic field** of the given magnet or magnetic configuration.



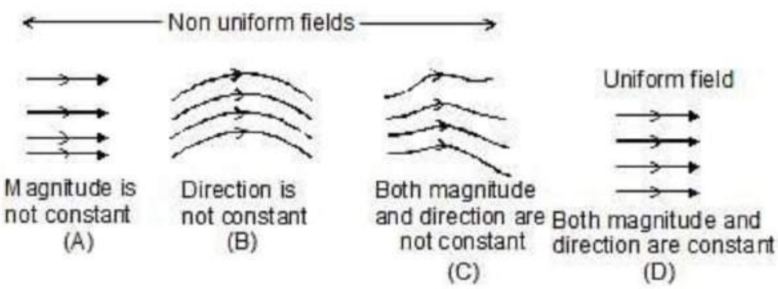
Bar-magnet (A)   U-Shape magnet (B)

In order to visualize a magnetic field graphically, **Michael Faraday** introduced the concept of lines.
According to him a line is an imaginary curve the tangent to which at a point gives the direction of the field at that point.
Regarding magnetic field it is worth noting that :
(i) Outside a magnet, field are from north to to south pole while inside from south to north, i.e., magnetic lines are closed curves i.e., they appear to converge or diverge at poles.

(ii) The number of magnetic lines of field originating or terminating on a pole is proportional to its strength. μ_0 lines are assumed to be associated with a unit pole. so if a body encloses a pole of strength m, total lines linked with the body (called **magnetic flux**) will be $\mu_0(m)$.
(iii) Magnetic lines of field can never intersect each other because if they intersect at a point, intensity at that point will have two directions which is absurd.
(iv) Magnetic lines of field have a tendency to contract longitudinally like a stretched elastic string (producing attraction between opposite poles) and repel each other laterally (resulting in repulsion between similar poles)



(v) Number of lines of field per unit area, normal to the area at a point, represents the magnitude of field at that point. so crowded lines represent a strong field while distant lines represent weak field. Further, if the lines of force are equidistant and straight the field is uniform otherwise not



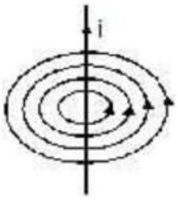
(vi) In a region of space where there is no magnetic field, there will be no lines of field. This is why, at a **neutral point** (where resultant field is zero) there cannot be any line of field.
(vii) Magnetic lines of field originate from or enter in the surface of a magnetic material at any angle.
(viii) Magnetic lines of field exist inside every magnetised material
(ix) As mono-poles do not exist, the total magnetic flux linked with a closed surface is always zero, i.e.,

$$\oint \vec{B}.\vec{ds} = \mu_0(0) = 0$$

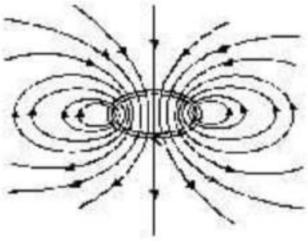
This law is called Gauss's law for magnetism.

Magnetic field line due to some important structure

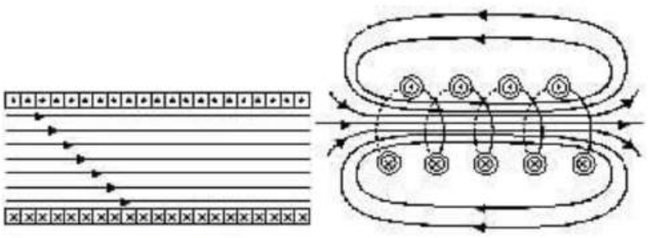
1. Straight current carrying wire



2. Circular coil



3. Solenoid



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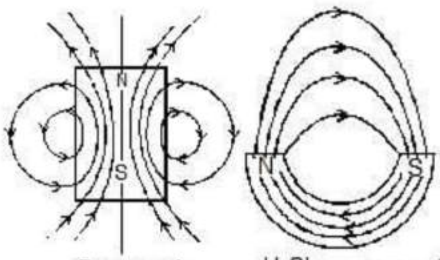

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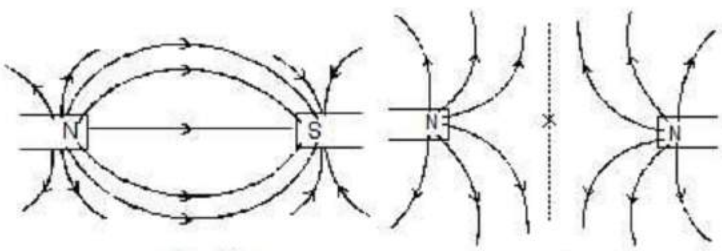

Attraction
(A)
Repulsion
(B)

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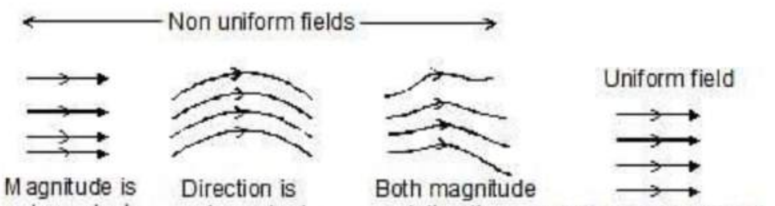

Magnitude is not constant (A)
Direction is not constant (B)
Both magnitude and direction are not constant (C)
Both magnitude and direction are constant (D)

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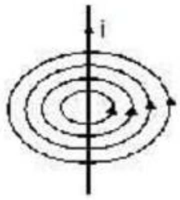

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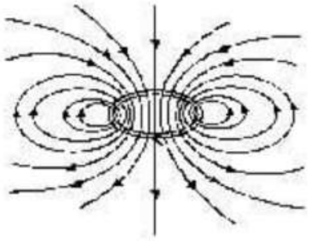

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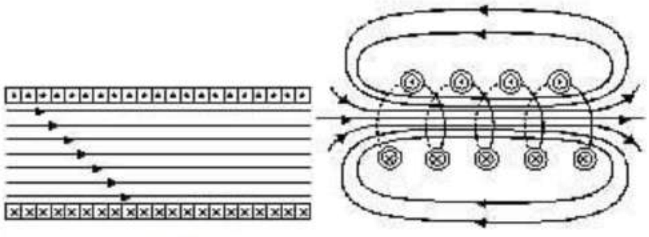


(ideal) (Real)

Magnet & Magnetic Field

16. Magnet:

16.1 Pole strength, magnetic dipole and magnetic dipole moment:

A magnet always has two poles `N' and `S' and like poles of two magnets repel other and the unlike poles of two magnets attract each other they form action reaction pair.

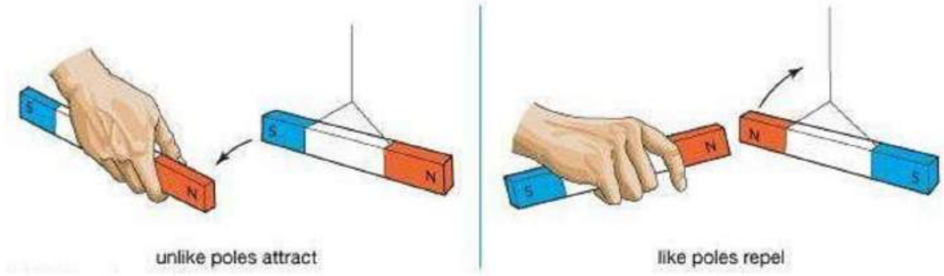
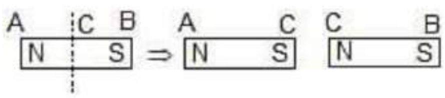
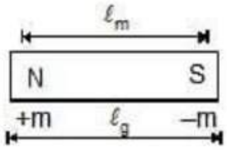


Fig: Magnetic poles

The poles of the same magnet do not comes to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. SO, `N' and `S' always exist together.
Therefore, they are:



Known as +ve and -ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as -ve pole (or -ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" + m and - m respectively (just like we have charge +q and -q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also). A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges -q and +q). It is called MAGNETIC DIPOLE and it has a direction is from -m to +m that means from `S' to `N').



$M = m.l_m$ here l_m = magnetic length of the magnet. l_m is slightly less than l_g (it is geometrical length of the magnet = end to end distance). The `N' and `S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m / \ell_g \simeq$ 0.84].
The units of m and M will be mentioned afterwards where you can remember and understand.

16.2 Magnetic field and strength of magnetic field:

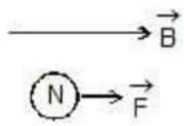
The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called "MAGNETIC FORCE". This field is quantitatively represented by "STRENGTH OF MAGNETIC FIELD" or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by. It is a vector quantity.

Definition of $\vec{B}$: The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

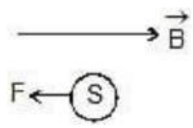
Mathematically, $\vec{B} = \frac{\vec{F}}{m}$

Here = magnetic force on pole of pole strength m. m may be +ve or -ve and of any value. S.I. unit of is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).

We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on -ve pole (south pole) it will be opposite to the direction of $\vec{B}$.



and

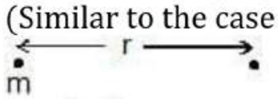


The field generated by sources does not depend on the test pole (for its any value and any sign).

(A) $\vec{B}$ due to various sources:

(i) Due to a single pole:

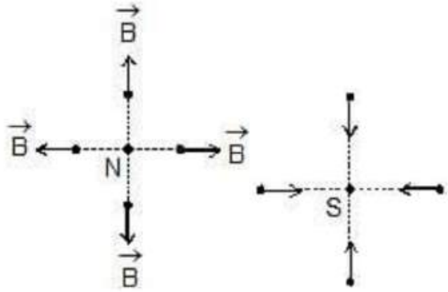
(Similar to the case of a point charge in electrostatics)



$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}$ .....(23)

This is magnitude

Direction of B due to north pole and due to south poles are as shown.

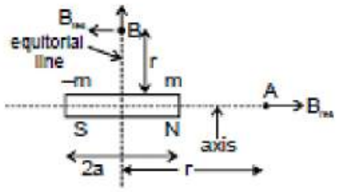


in vector from $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$ ...(24)

here m is with sign and = position vector of the test point with respect to the pole.

(ii) Due to a bar magnet:

(Same as the case of electric dipole in electrostatics) independent case never found. Always `N' and `S' exist together as magnet.

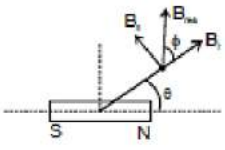


at A (on the axis) = $2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r ...(25)

at B (on the equatorial) = - $\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r ...(26)

At General point:

$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$

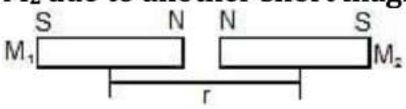


$B_n = \left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$

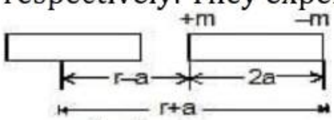
$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$ ...(27 (a))

$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$ ...(28 (b))

Ex. 36: Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



Ans: To find the magnetic force we will use the formula of `B' due to a magnet. We will also assume m and -m as pole strengths of `N' and `S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and -m respectively. They experience magnetic forces F_1 and F_2 as shown.



$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$ and $F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$

Therefore, F_{res} F_1-F_2=$2\left(\frac{\mu_0}{4\pi}\right) M_1 m \left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$

$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3}-\left(1+\frac{a}{r}\right)^{-3}\right]$

By using, Binomial expansion, and neglecting terms of high power we get

$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$

$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4} = 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$

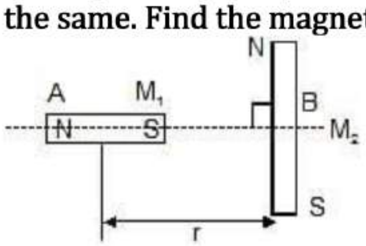
Direction of F_{res} is towards right.

Alternative Method:

$B = \frac{\mu_0}{4\pi}\frac{2M_1}{r^3} \Rightarrow \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$

$F = -M_2 \times \frac{dB}{dr} \Rightarrow F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1 M_2}{r^4}$

Ex. 37: Two short magnets A and B of magnetic dipole moments M_1 and M_2 respectively are placed as shown. The axis of `A' and the equatorial line of `B' are the same. Find the magnetic force on one magnet due to the other.



Ans: $F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2M_1}{r^4}$ upwards on M_1

down wards on M_2

Ex 38. A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Ans: The pole strength is m = 120
CGS units = 12 A-m
Magnetic length is $2l = 10$ cm or $l = 0.05$ m
Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)]^2-(0.05m)^2]^2} = 3.4\times10^{-5}\text{ T.}$$

Ex. 39: Find the magnetic field due to a dipole of magnetic moment 1.2 A-m² at a point 1 m away from it in a direction making an angle of 60° with the dipole-axis.
Ans: The magnitude of the field is

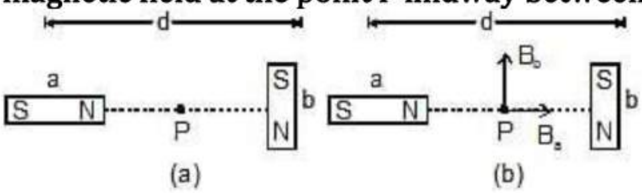
$$B = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{12A-m^2}{1m^2}\sqrt{1+3\cos^2 60^\circ}$$

The direction of the field makes an angle a with the radial line where

$$\tan a = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

Ex. 40: Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



Ans: The point p is in end-on position for the dipole a and in broadside-on position for the dipole b'. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore

$B = \sqrt{B_a^2 + B_b^2}$

$= \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2 + 2^2}$

$= \frac{2\sqrt{5}\mu_0 M}{\pi d^2}$

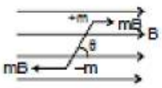
The direction of this field makes an angle a with B_a such that tan a = B_b/B_a = 1/2.

16.3 Magnet in an external uniform magnetic field:

(same as case of electric dipole)

F_{res} = 0 (for any angle)

t = MB sin θ



* here θ is angle between $\vec{B}$ and $\vec{M}$

Note:

$\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$

$\vec{\tau}$ is same about every point of the dipole it's potential energy is U = - MB cosθ = - $\vec{M}.\vec{B}$

θ= 0° is stable equilibrium

θ = p is unstable equilibrium

for small `q' the dipole performs SHM about q = 0° position

τ = - MB sin θ;

I α = - M B sin θ

$\alpha = -\left(\frac{MB}{I}\right)\theta$

for small θ, sin θ ≃ θ

Angular frequency of SHM

$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \Rightarrow$ **T = 2p** $\sqrt{\frac{I}{MB}}$

here I = I_{cm} if the dipole is free to rotate

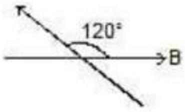
= I_{hinge} if the dipole is hinged

***Ex.* 41: A bar magnet having a magnetic moment of 1.0 × 10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field B = 4 × 10^{-5}T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.**

Ans: The work done by the external agent = change in potential energy

$= (-MB \cos\theta_2) - (-MB \cos\theta_1)$
$= - MB (\cos 60° - \cos 0°)$
$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^4 \text{ J/T}) (4 \times 10^{-5} \text{ T}) = 0.2 \text{ J}$

Ex. 42: A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure.



Find:
(i) Its kinetic energy at $\theta = 90°$
(ii) its maximum kinetic energy during the motion.
(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

Ans: (i) Apply energy conservation at $\theta = 120°$ and $\theta = 90°$
$= - MB \cos 120° + 0$
$= - MB \cos 90° + (K.E)$
$KE = \frac{MB}{2}$ **Ans.**
(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0º$.
Now apply energy conservation between $\theta = 120º$ and $\theta = 0º$.
$= -mB \cos 120º + 0$
$= - mB \cos 0º + (KE)_{max}$
$(KE)_{max} = \frac{3}{2}MB$ **Ans.**
The K.E. is max at $\theta = 0°$ can also be proved by torque method. From $\theta = 120°$ to $\theta = 0°$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0°$. Beyond that t reverses its direction and then K.E. starts decreasing. Therefore, $\theta = 0°$ is the orientation of M to here the maximum K.E.
(iii) Since `q' is not small.
Therefore, the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120°.

Ex. 43: A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes p/2 seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.
(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Ans: (a) The moment of inertia of the magnet about the axis of rotation is

$I = \frac{m'}{12}(L^2 + b^2)$

$= \frac{100 \times 10^{-3}}{12}$ $[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]$ kg-m^2 = $\frac{25}{6} \times 10^{-5}$ kg − m^2

We have, $T = 2\pi\sqrt{\frac{I}{MB}}$

or, $M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5} \text{kg/m}^2}{6 \times (25 \times 10^{-6} T) \times \frac{\pi^2}{4} s^2}$ = 27 A-m^2

(b) In this case the moment of inertia becomes

$I' = \frac{m'}{12}(L^2 + b^2)$ where b' = 0.5 cm.

The time period would be

$T' = \sqrt{\frac{I'}{MB}}$ ...(ii)

Dividing by equation (i),

$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}} = \frac{\sqrt{(7cm)^2 + (0.5\,cm)^2}}{\sqrt{(7\,cm)^2 + (1.0\,cm)^2}}$ = 0.992

or, $T' = \frac{0.992 \times \pi}{2} s$ = 0.496 p s.

16.4 Magnet in an External Non-uniform Magnetic field:

No special formula are applied is such problems. Instead see the force on individual poles and calculate the resultant force torque on the dipole.

Note:

- Force due to Non-uniform Magnetic field:

$F = -M\frac{dB}{dr}$

- If a source of Magnetic Moment $\vec{M}$ have dimension very less than the distance of point of application then we can replace it with magnet of magnetic moment equal to $\vec{M}$.

17. TERRESTRIAL MAGNETISM:

Earth is a natural source of magnetic field.

17.1 Elements of the Earth's Magnetic Field:

The earth's magnetic field at a point on its surface is usually characterised by three quantities:
(a) declination
(b) inclination or dip and
(c) horizontal component of the field. These are known as the elements of the earth's magnetic field.

(a) Declination:

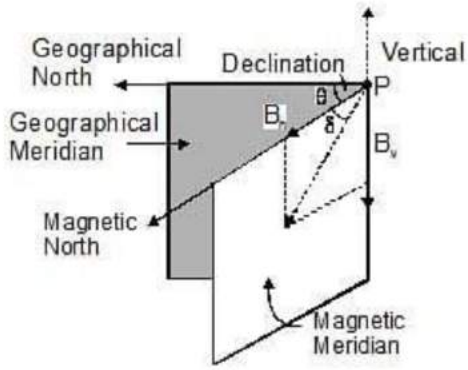
A plane passing through the geographical poles (that is, through the axis of rotation of the earth) and a given point P on the earth's surface is called the geographical meridian at the point P. Similarly, the plane passing through the geomagnetic poles (that is, through the dipole-axis of the earth) and the point P is called the magnetic meridian at the point P.
The angle made by the magnetic meridian at a point with the geographical meridian is called the declination at that point.

(b) Inclination or dip:

The angle made by the earth's magnetic field with the horizontal direction in the magnetic meridian, is called the inclination or dip at that point.

(c) Horizontal component of the earth's magnetic field:

As the name indicates, the horizontal component is component of the earth's magnetic field in the horizontal direction in the magnetic meridian. This direction is towards the magnetic north.
Figure shows the three elements. Starting from the geographical meridian we draw the magnetic meridian at an angle q (declination). In the magnetic meridian we draw the horizontal direction specifying magnetic north. The magnetic field is at an angle d (dip) from this direction. The horizontal component B_H and the total field B are related as
$B_H = B \cos \delta$
or, $B = B_H / \cos \delta$



Thus, from the knowledge of the three elements, both the magnitude and direction of the earth's magnetic field can be obtained.

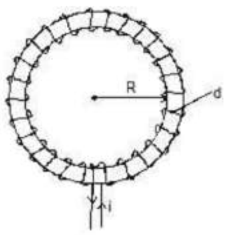
Ex. 45 The horizontal component of the earth's magnetic field is 3.6×10^{-5} T where the dip is 60º. Find the magnitude of the earth's magnetic field.

Ans: We have $B_H = B \cos \delta$

$$B = \frac{B_H}{\cos\delta} = \frac{3.6 \times 10^{-5} T}{\cos 60^\circ} = 7.2 \times 10^{-5} T$$

Toroid & Solenoid

9. Toroid: It is on hollow circular tube have windings of conducting wire closely attached to each other circularly on it (as shown below)



for ideal Toroid d << R

Magnetic field in Toroid

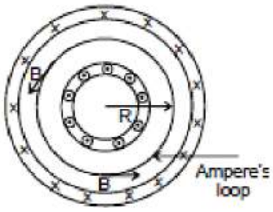
Let N = Total No. of turns

Now from Ampere's circuital law

$$\oint B.d\ell = \mu_0 i_{in}$$

$B.2\pi R = \mu_0 \, i_{in} = \mu_0 \, Ni$

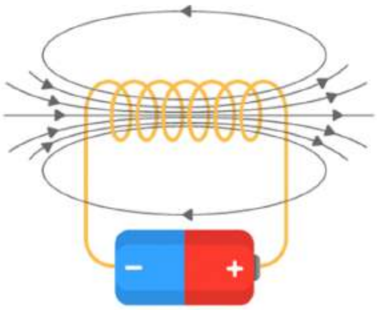
$\Rightarrow B = \frac{\mu_0 Ni}{2\pi R}$



$n = \frac{N}{2\pi R}$ = No of turns per unit length
so $B = \mu_0 n i$

What is Solenoid?

Let us consider a solenoid, such that its length is large as compared to its radius. Here, the wire is wound in the form of the helix with a very little gap between any two turns. Also, the wires are enameled, thus rendering them insulated from each other. As a result, each turn can be taken as a closed circular loop. The magnetic field thus generated is equivalent to that generated by a circular loop and the total magnetic field generated by the solenoid can be given as the vector sum of force generated by each such turn. The magnetic field lines generated inside a finite solenoid has been shown in the figure below.



Solenoid

We can see from the figure that the magnetic field inside the solenoid is uniform in nature and is along the axis of the solenoid. The field at the exterior at any point immediately to the solenoid is very weak and the field lines cannot be seen near the close vicinity. It is important to note that the field inside it is parallel to its axis at every position.

From the Ampere's Law, the magnetic force produced by a solenoid can be given as,

$F = \mu_0 n I$

Where n is the number of turns of the wire per unit length, I is the current flowing through the wire and the direction is given using the right-hand thumb rule.

10. Infinite Current Carrying sheet

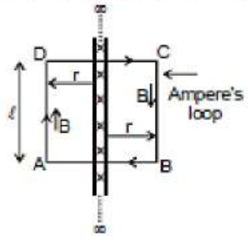
Now from Ampere's loop

$\oint B.d\ell = \mu_0 \lambda \ell$

$\int_{AB} B.d\ell + \int_{BC} B.d\ell + \int_{CD} B.d\ell + \int_{DA} B.d\ell = \mu_0 \lambda \ell$

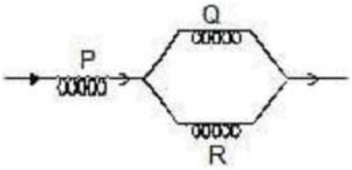
$B\int d\ell + \int B' d\ell \cos 90° + B\int d\ell + \int B' d\ell \cos 90° = \mu_0 \lambda \ell$

$Bl + 0 + Bl + 0 = \mu_{0\lambda}\, l$



$B = \frac{\mu_0 \lambda}{2}$

<u>Ex.14</u> Three identical long solenoids P, Q and R are connected to each other as shown in figure. if the magnetic field at the center of P is 2.0 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.



Sol.

As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0\, ni$. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 1.0 T

11. Magnetic force on moving charge

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula:

$\vec{F} = q(\vec{v} \times \vec{B})$ Put q with sign. ...(9)

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

$\vec{B}$

11.1 DIFFERENCE BETWEEN MAGNETIC FORCE AND ELECTRIC FORCE

(1) Magentic force is always perpendicular to the field while electric force is collinear with the field.

(2) Magnetic force is velocity dependent, i.e., acts only when the charged particle is in motion while electric force (qE) is independent of the state of rest or motion of the charged particle.
(3) Magentic force does no work when the charged particle is displaced while the electric force does work in displacing the charged particle.

Note:

$\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
Therefore, $\vec{F} \perp \vec{v}$ Therefore, power due to magnetic force on a charged particle is zero. (use the formula of power P = $\vec{F} \cdot \vec{v}$ for its proof)
Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion.
The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
On a stationary charged particle, magnetic force is zero.
If $\vec{v} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Ex.15 A Charged particle of mass 5 mg and charge q = +2μC has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m² respectively.

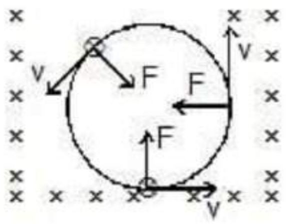
Sol. $\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6} (2\hat{i} - 3\hat{j} + 4\hat{k}) \times (3\hat{j} - 2\hat{k})$

By Newton's Law $\vec{a} = \frac{\vec{F}}{m} = \frac{2 \times 10^{-6}}{5 \times 10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$
$= 0.8\,(-3\hat{i} + 2\hat{j} + 3\hat{k})\ m/s^2$

Ex.16 A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.
Sol. $\because \vec{F} \perp \vec{B}$
Therefore, $\vec{a} \perp \vec{B}$
Therefore, $\vec{a} \cdot \vec{B} = 0$
Therefore, $(2\hat{i} + x\hat{j}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0$
$\Rightarrow -6 + 2x = 0 \Rightarrow x = 3.$

12. MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD.
12.1 WHEN THE CHARGED PARTICLE IS GIVEN VELOCITY PERPENDICULAR TO THE FIELD

Let a particle of charged q and mass m is moving with a velocity v and enters at right angles to a uniform magnetic field $\vec{B}$ as shown in figure.
The force on the particle is qvB and this force will always act in a direction perpendicular to v. Hence, the particle will move on a circular path. If the radius of the path is r then



$\frac{mv^2}{r} = Bqv$ or, $r = \frac{mv}{qB}$ ...(10)
Thus, radius of the path is proportional to the momentum mv of the particle and inversely proportional to the magnitude of magnetic field.

Time period: The time period is the time taken by the charged particle to complete one rotation of the circular path which is given by,

$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$ ...(11)

The time period is independent of the speed v.

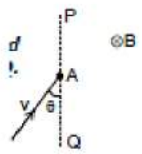
Frequency: The frequency is number of revolution of charged particle in one second, which is given by,
$\nu = \frac{1}{T} = \frac{qB}{2\pi m}$ ..(12)
and angular frequency = $\omega = 2\pi\nu$
<u>Ex 17</u>. **A proton (p), a - particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).**

Sol. $R = \frac{\sqrt{2mK}}{qB}$

Therefore, $R_p : R_a : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{qB} : \frac{\sqrt{2.2mK}}{qB}$
$= 1 : 1 : \sqrt{2}$
$T = 2\pi m/qB$

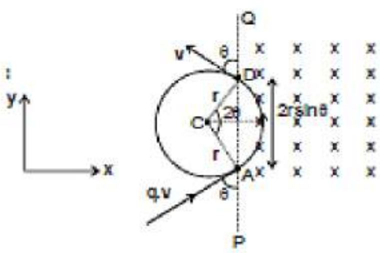
Therefore, $T_p : T_a : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB}$
= 1: 2: 2 **Ans.**

<u>Ex.18</u> A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ. Find



(i) time spent,
(ii) distance travelled in the magnetic field
(iii) impulse of magnetic force.

Sol. The particle will move in the field as shown. Angle subtended by the arc at the centre = 2q
(i) Time spent by the charge in magnetic field
wt = q $\Rightarrow \frac{qB}{m} t = \theta \Rightarrow t = \frac{m\theta}{qB}$
(ii) Distance travelled by the charge in magnetic field:
$= r(2\theta) = \frac{mv}{qB} 2\theta$



(iii) Impulse = change in momentum of the charge
$= (-mv \sin\theta + mv \cos\theta\ \hat{j}) - (mv \sin\theta\ \hat{i} + mv \cos\theta) = -2mv \sin\theta\ \hat{i}$

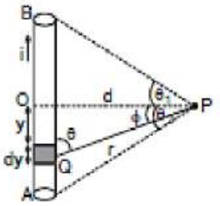
Force on Current Carrying Conductor

3. FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE
3.1 WHEN THE WIRE IS OF FINITE LENGTH

Consider a straight wire segment carrying a current i and there is a point P at which magnetic field to be calculated as shown in the figure. This wire segment makes angle θ_1 and θ_2 at that point with normal OP. Consider an element of length dy at a distance y from O and distance of this element from point P is r and line joining P to Q makes an angle q with the direction of current as shown in figure. Using Biot-Savart Law magnetic field at point P due to small current element is given by

$$dB = \frac{\mu_0 i}{4\pi}\left(\frac{dy \sin\theta}{r^2}\right)$$

As every element of the wire contributes to $\vec{B}$ in the same direction, we have



$$B = \frac{\mu_0 i}{4\pi}\int_A^B \frac{dy \sin\theta}{r^2}$$ ....(i)

From the triangle OPQ as shown in diagram, we have

$y = d \tan\phi$

or $dy = d \sec^2\phi \, d\phi$

and is same triangle,

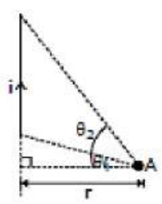
$r = d \sec\phi$ and $q = (90^\circ - \phi)$, where ϕ is angle between line OP and PQ

Now equation (i) can be written in this form

$$\therefore B = \frac{\mu_0}{4\pi}\frac{i}{d}\int_{-\theta_2}^{\theta_1} \cos\phi \, d\phi$$

or $B = \frac{\mu_0 i}{4\pi d}[\sin\theta_1 + \sin\theta_2]$ ...(3)

Note:θ_1 &θ_2 mustbetakenwithsign

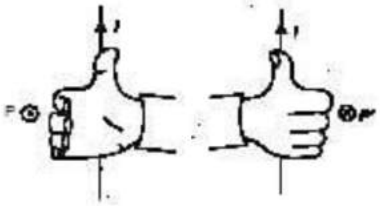


For the case shown in figure

B at A = $\frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1) \otimes$

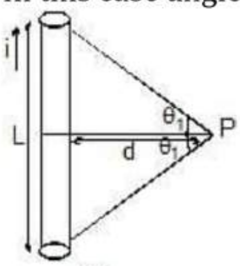
Direction of $\vec{B}$: The direction of magnetic field is determined by the cross product of the vector $id\vec{l}$ with $\vec{r}$. Therefore, at point P, the direction of the magnetic field due to the whole conductor will be perpendicular to the plane of paper and going into the plane.

Right-hand Thumb Rule: The direction of B at a point P due to a long, straight wire can be found by the right-hand thumb rule. The direction of magnetic field is perpendicular to the plane containing wire and perpendicular from the point. The orientation of magnetic field is given by the direction of curl fingers if we stretch thumb along the wire in the direction of current. Refer figure.



Conventionally, the direction of the field perpendicular to the plane of the paper is represented by $\otimes$ if into the page and by $\odot$ if out of the page.
Now consider some special cases involving the application of equation (3)

Case 1: When the point P is on the perpendicular bisector
In this case angle $\theta_1 = \theta_2$, using result of equation (3), the magnetic field is

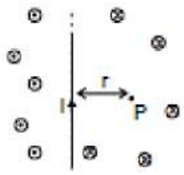


$B = \frac{\mu_0}{4\pi}\frac{2i}{d}\sin\theta_1$

where $\sin\theta_1 = \frac{L}{\sqrt{L^2 + 4d^2}}$

Case - 2
(i) If the wire is infinitely long then the magnetic field at `P' (as shown in the figure) is given by (using $q_1 = q_2 = 90°$ and the formula `B' due to straight wire)

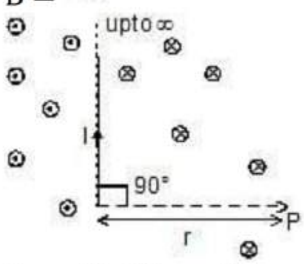
$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \mu \frac{I}{r}$



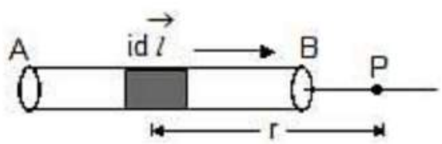
The direction of $\vec{B}$ at various is as shown in the figure. The magnetic lines of force will be concentric circles around the wire (as shown earlier)

(ii) If the wire is infinitely long but `P' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At `P'

$B = \frac{\mu_0 I}{4\pi r}$



Case III: When the point lies along the length of wire (but not on it)

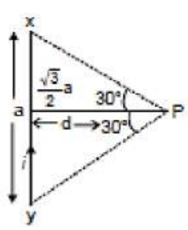


If the point P is along the length of the wire (but not one it), then as $d\vec{l}$ and $\vec{r}$ will either be parallel or antiparallel, i.e., q = 0 or p, so $i\,d\vec{l} \times \vec{r} = 0$ and hence using equation (1)

$\vec{B} = \int_A^B d\vec{B} = 0$

<u>Ex-1</u> Calculate the magnetic field induction at a point distance, $\frac{a\sqrt{3}}{2}$ metre from a straight wire of length `a' metre carrying a current of i amp. The point is on the perpendicular bisector of the wire.

Sol. $B = \frac{\mu_0}{4\pi}\frac{i}{d}$ [sinq$_1$ + sinq$_2$]

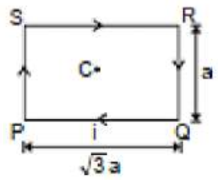


$= 10^{-7}\left[\frac{i}{(a\sqrt{3}/2)}\left(\frac{1}{2}+\frac{1}{2}\right)\right]$

$= \frac{2i}{a\sqrt{3}} \times 10^{-7}\,T$

Perpendicular to the plane of figure (inward).

<u>Ex.2</u> Find resultant magnetic field at `C' in the figure shown.



Sol. It is clear that `B' at `C' due all the wires is directed Ä. Also B at `C due PQ and SR is same. Also due to QR and PS is same

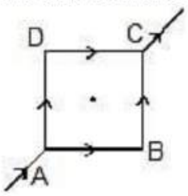
Therefore, $B_{res} = 2(B_{PQ} + B_{SP})$

$B_{PQ} = (\sin 60° + \sin 60°)$

$B_{SP} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}\,a}{2}} (\sin 30° + \sin 30°)$

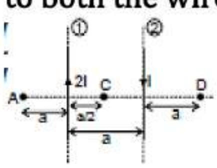
$\Rightarrow B_{res} = 2\left(\frac{\sqrt{3}\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$

<u>Ex.3</u> Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.



Sol. The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

<u>Ex.4</u> In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents 2 I and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned.



Find (i)* $\vec{B}$ *at A, C, D

(ii) position of point on line A C D where $\vec{B}$ is zero.

Sol. (i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

at A : $\vec{B}_1$ is ⊙ and $\vec{B}_2$ is ⊗

$B_1 = \frac{\mu_0 2I}{2\pi a}$ and $B_2 = \frac{\mu_0 I}{2\pi 2a}$

Therefore, $B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a}$ ⊙ **Ans.**

at C: $\vec{B}_1$ is ⊗ and $\vec{B}_2$ also ⊗

Therefore, $B_{res} = B_1 + B_2 =$

$\frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a}$ ⊗ **Ans.**

at D : $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is ⊙ and both are equal in magnitude

Therefore, $B_{res} = 0$ **Ans.**

(ii) It is clear from the above solution that B = 0 at point `D'.